Experimental study of the evolution of a velocity perturbation in fully developed turbulence

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This Brief Report concerns an experimental analysis of the interaction between a fully developed turbulent flow and a perturbation generated in the physical domain. A coherent averaging technique permits one to extract information on the response function of the turbulent flow and to insulate only effects related to interactions between the background turbulence and the perturbation. It is shown that the linear approximation of the impulse response function obtained from Kraichnan's direct interaction approximation theory is in agreement with the present results. [S1063-651X(97)09308-2]

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Turbulence can be considered as a dynamical system involving a great number of degrees of freedom [1] whose dynamics is complicated by the nonlinear nature of interactions among different modes. Therefore, the study of the dynamics of turbulent velocity fields can be conducted following different approaches, but always in approximate manners. In particular, as for dynamical systems, the study of turbulence can be conducted by means of an analysis of the response of the velocity field to an appropriate perturbation. This approach permits the knowledge of response functions, interscale dynamics, and interactions between different scales, which are fundamental aspects for turbulent models, theories, and flow control problems.

Early theoretical work concerning the effect of a perturbation on turbulent flows was performed by Lin [2] based on a spectral energy transfer analysis. Subsequently, Townsend [3] analyzed the problem in the physical domain by considering a turbulent field subjected to small variations of the mean velocity. The direct interaction approximation (DIA) theory, introduced by Kraichnan [4], is also based on the determination of the impulse-response tensor of a fully developed turbulent velocity field in homogeneous and isotropic conditions. This theory leads to a closed set of equations that governs the dynamics of velocity fluctuations. Of particular interest is the linear approximation of the response function, based on the assumption of a Gaussian probability distribution of the velocity field. The approximated linear response function $g(k,\tau)$ can be written as [4,5]

$$g(k,\tau)=e^{-(1/2)u_0^2k^2\tau^2}, \tag{1}$$

where u_0 is the velocity rms, k represents the wave number, and τ is the time. This relation applies for homogeneous turbulence and a range of scales sufficiently smaller than the perturbation injection wave number. A similar conclusion for the turbulent diffusion of a passive scalar has been reached in Ref. [12] for wave vectors in the inertial range. The experimental validation of Eq. (1) is one of the primary aspects that motivated the present work.

From an experimental viewpoint, Kellog and Corrsin [7] and Itsweire and Van Atta [8] analyzed spectrally localized disturbances in grid-generated turbulence. They observed an exponential decay of the spectral line corresponding to the perturbation characteristic scale. Qualitative agreement with the DIA prediction given in Eq. (1) also has been documented [7]. The evolution of a perturbation generated in the physical space has been analyzed only in convective flows [9]. Specifically, these studies were based on an experimental analysis of the evolution of a thermal pulse, in strong spatiotemporal chaotic regimes of Rayleigh-Benárd convection in an annulus. As far as we know, neither experiments nor numerical simulations have been devoted so far to the analysis of the evolution of a perturbation generated in the physical domain and moving through a fully developed turbulent velocity field, although this approach appears to be of interest for a direct characterization of turbulence dynamics. The study of this subject from an experimental point of view is the main purpose of the present paper. Furthermore, the present analysis permits the experimental validation of a DIA response function prediction. In the present experiment the perturbation is actually a velocity step generated in the physical domain, which interacts with a turbulent axisymmetric jet. The choice of a *step function* has been dictated by the need of a perturbation spatially localized in addition to the need of precise knowledge of the perturbing energy spectrum. Since a Dirac δ function cannot be realized in experiments with sufficient accuracy, the step function appears to be the most appropriate function to be used as external forcing. Furthermore, its energy spectrum, apart oscillations, is characterized by a power-law decay of the form k^{-2}, which is sufficiently different from the Kolmogorov [10] $k^{-5/3}$ shape of the spectra, to be expected in fully developed turbulence. Measurements were conducted with a single probe hot wire anemometer (probe TSI 1260 of length $l_w=500$ μm) in four axial positions x_i downstream of a jet of diameter $D=120$ mm. Specifically, the measurement points x_i/D, with $i=1,\ldots,4$, were equal to 12, 16, 19, and 22, respectively. The analysis in the wave-number domain is conducted, from the data acquired at the fixed points x_i, by the use of the Taylor hypothesis to convert time in space, that is, $k=2\pi f/V_0$, where f is the frequency and V_0 is the mean velocity of the flow. Actually we used a *local* and more appropriate application of the Taylor hypothesis [11],

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which takes into account the fluctuations of the large-scale velocity by a suitable sampling technique of the velocity signal. In this way the analysis in the wave-number domain can be conducted safely when the turbulence rate is very high, which is the case for the jet flow, and even when sharp variations of the mean velocity are observed.

The jet was mounted inside a low-speed wind tunnel with very low turbulence level (0.3%). Two different flow conditions were considered. The first case was achieved using only the wind tunnel almost laminar flow without the jet. This condition will be referred to as the *laminar* one. In the second case the jet nozzle was inserted into the wind tunnel. Indicating by $u_j(x_i,t)$ the fluctuating turbulent velocity generated by the jet and by u_j' the rms of $u_j(x_i,t)$, we had $u_j'/V_0 \simeq 25\%$, which is typical for jet turbulence. Therefore, the case with jet turbulence will be referred to as the *turbulent* experiment. The perturbation has been generated by a small jet, coaxial with the large one, with diameter d much smaller than D. Two different d were considered: 5 and 10 mm. The small jet was driven by compressed air, which was switched on and off by an electrically controlled valve with a response time of 0.1 sec. The opening of the valve was controlled by the data acquisition system. It has been checked that, at the outflow of the small jet, the velocity signal was a step function characterized by a sufficiently steep slope corresponding to a scale one order of magnitude smaller than the integral scale of the turbulent flow. Indicating by $u_{p1}(x_i,t)$ and $u_{p2}(x_i,t)$ the perturbation velocities in the laminar (subscript 1) and the turbulent (subscript 2) experiments, the total velocity $V(x_i,t)$ achieved in the different flow conditions can be formalized as follows: in the laminar experiment $V_1(x_i,t)=V_0+u_{p1}(x_i,t)$ and in the turbulent experiment $V_2(x_i,t)=V_0+u_j(x_i,x)+u_{p2}(x_i,x)$. Notice that in this notation the interaction of the perturbation with turbulence manifests itself by the fact that $u_{p1}\neq u_{p2}$. Furthermore, we indicate with S_0 the original spectrum of the perturbation before the interaction with the turbulent field. As pointed out above, S_0, in our experiment, has the form $S_0(k)\sim k^{-2}$. The spectrum of the perturbation $S_{p,m}(x_i,k)$ at a position x_i can then be written as

$$S_{p,m}(t_i,k)=G_m(t_i,k)S_0(k), \tag{2}$$

where $t_i=x_i/V_0$ and $m=1,2$ depending on the *background* flow ($m=1$ is the laminar case and $m=2$ is the turbulent case). The aim of the present work is to analyze the relaxation in time of the *excited* modes, which corresponds to analyzing the linear response function $G_m(t_i,k)$. This function is the response of the background turbulent flow to a δ-function perturbation. Conversely, $G_m(t_i,k)$ can be viewed as the deformation to the perturbation spectrum induced by the background turbulent field. Furthermore, we recall that, by definition [13], the response function G_m has meaning only if it is obtained as an ensemble average over many configurations of the background turbulent flow subjected to the same perturbation.

In the laminar experiment, V_0 was of about 8 m/s. In the turbulent case, two different large jet outflow velocities have been analyzed, 30 and 15 m/s, corresponding, at the measuring locations, to a velocity V_0 of the order of 8 and 2.5 m/s respectively. The corresponding Re_λ (that is, the turbulent

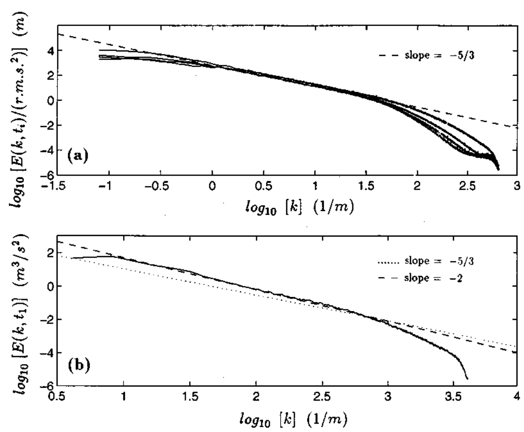

FIG. 1. (a) Wave-number spectra of $u_j(x_i,t)$ normalized with respect to the local velocity rms and measured at four different positions. The straight line represents the slope $-5/3$. (b) Wave-number spectrum of $V_1(x_1,t)$. The dashed line represents the slope -2 and the dotted line the slope $-5/3$.

Reynolds number based on the Taylor microscale and on the velocity fluctuation rms) varied from about 280 at x_1 to about 200 at x_4 for the lowest mean velocity and from about 320 at x_1 to about 250 at x_4 for the highest mean velocity.

The turbulent field $u_j(x_i,t)$ was preliminary qualified by a spectral analysis. In Fig. 1(a) the wave-number energy spectra normalized with respect to the velocity rms and calculated in the four measurement positions are presented for the case at $V_0=2.5$ m/s. The collapse of the curve is satisfactory apart the smallest scales and the inertial range is evidenced by the $-5/3$ slope in the log-log representation. This indicates that the jet turbulence $u_j(x_i,t)$ can be considered as *fully developed* and that the wake of the small coaxial jet, which partially obstructs the larger jet orifice, actually does not affect the background velocity field.

The aim of the laminar experiment was to analyze the evolution of the perturbation in the absence of background turbulence. In Fig. 1(b) the spectrum of $V_1(x_1,t)$ is presented and the expected k^{-2} behavior is observed. In the laminar case it is found that the perturbation is only convected by the mean flow and diffused by viscosity. More specifically, considering the other positions x_i, it is observed that the function $G_1(t_i,k)$ [see Eq. (2)] is of the form $\sim\exp(-2t_ik^2\nu)$, where ν is the kinematic viscosity and $t_i=x_i/V_0$. This means that in the laminar case, the spectrum evolution is controlled by pure diffusive effects.

When $V_2(x_i,x)$ is considered, the perturbation $u_{p2}(x_i,t)$ can hardly be observed because it has an amplitude comparable to the largest fluctuations of $u_j(x_i,t)$. As an example we show in Figs. 2(a) and 2(b) the velocity signals $V_2(x_i,t)$ at x_1 and x_2 and $V_0=8$ m/s. The perturbation cannot be clearly observed. Therefore, the evolution of the perturbation is recovered by means of coherent averages of the velocity signals. To do that, the velocity signal acquisitions and the perturbation injection are activated simultaneously since they are commanded by the same trigger pulse generated by the data acquisition system. Acquisitions are then conducted in phase for a duration of about 2 s with a frequency sampling of 8 kHz. The sampling time corresponds, in terms of spatial length, approximately to the probe size.

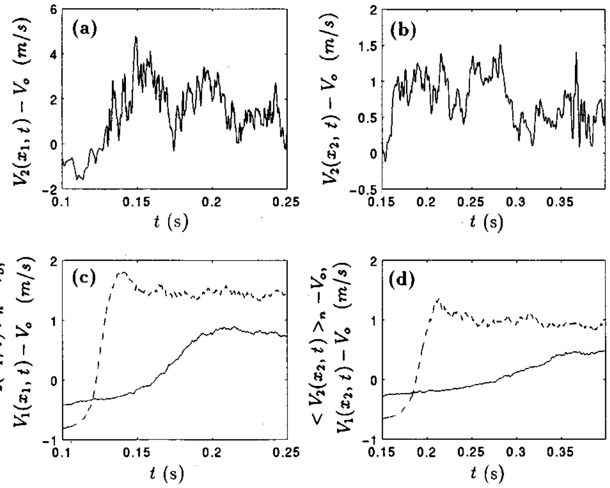

FIG. 2. (a) $V_2(x_1,t)$ and (b) $V_2(x_2,t)$. We see clearly that the perturbation cannot be observed as it has the same amplitude as the velocity fluctuations. Also shown a comparison between $V_1(x_i,t)$ (dashed line) and $\langle V_2(x_i,t)\rangle_n$ (solid line) for (c) x_1 and (d) x_2.

The total amount of acquired files ranges from 300 to 500 depending on the turbulence levels at the different positions and velocities. The ensemble averaging of the velocity signals synchronized with the injection of the perturbation yields the elimination of the background turbulent fluctuations characterized by randomly distributed phases. The averaging procedure can be written as

$$\langle V_2(x_i,t)\rangle_n = V_0 + \langle u_{p2}(x_i,t)\rangle_n$$

[since $\langle u_j(x_i,t)\rangle_n = 0$] , where $\langle\rangle_n$ indicates the ensemble averaging over n realizations. As it will be shown later, the term $\langle u_{p2}(x_i,t)\rangle_n$ retains the response property of the system as a result of the interaction of the perturbation with the turbulent field $u_j(x_i,t)$, produced by the big jet.

The velocities $\langle V_2(x_i,t)\rangle_n$ (for $V_0 = 8$ m/s) have been compared with the $V_1(x_i,t)$ measured in the laminar case. Obviously, in the laminar experiment, no average is needed in order for the perturbation to be evident. The comparison between $\langle V_2(x_i,t)\rangle_n$ and $V_1(x_i,t)$ is reported in Figs. 2(c) and 2(d) for x_1 and x_2, respectively. These figures show the effect of the coherent averages, which enhance the perturbation fronts and strongly reduce the background turbulence signal. Furthermore, we clearly observe that the front of the perturbation is steeper in the laminar case than in the turbulent one. This indicates that the shape of the perturbation is affected by the interaction with the background turbulence. More quantitative results can be achieved by the analysis of the step energy spectra in the different positions. First of all, accounting for the local Taylor hypothesis, it is found that in all cases the energy spectra no longer follow the k^{-2} power law. Second, by fixing a certain value of the wave vector k it is possible to analyze the evolution of a mode as a function of the distance from the jet or of the time delay $t_i = x_i/V_0$ needed by the perturbation to reach the measuring positions. It is found that in the turbulent cases the time evolution of modes amplitude follows an exponential law quadratic in time as $\sim\exp[-A(k)t_i^2]$, where $A(k)$ is a quadratic function of k, in contrast with the laminar case where the pure diffusive behavior (linear in t_i) is observed. This behavior is presented in Fig. 3(a) for wave numbers $k/(2\pi/D)$ ranging from 30 to 130. The plot is in log-linear scale and the x axis corresponds to t_i^2. Therefore, the observed linear trend clearly confirms that the decay law is an exponential quadratic in time. Analogous behavior is observed in the other cases (different mean velocities and different size d), which are not reported here. In order to check Eq. (1) further, we plotted previous data in nondimensional form and an example is reported in Fig. 3(b) for the same cases as in Fig. 3(a). Specifically, the energy magnitude is normalized with respect to its value at t_1, whereas the time is normalized with respect to u_0k, that is $t^* = u_0kt$. In this plot also the linear fits are reported. It has been found that for $k \gg k_0$, where k_0 is the injection wave number that is the inverse of the small jet diameter, the slope of the fits is close to -0.5. This result confirms that Eq. (1) correctly predicts the time evolution of the mode amplitudes. In contrast, the weak dependence on k of the slope of the fits of Fig. 4(b) indicates that the measured $A(k)$ tends exactly to $0.5(u_0k)^2$ of Eq. (1), only for $k \gg k_0$. The reasons for this discrepancy are still under investigation. It should be pointed out that similar forms of the

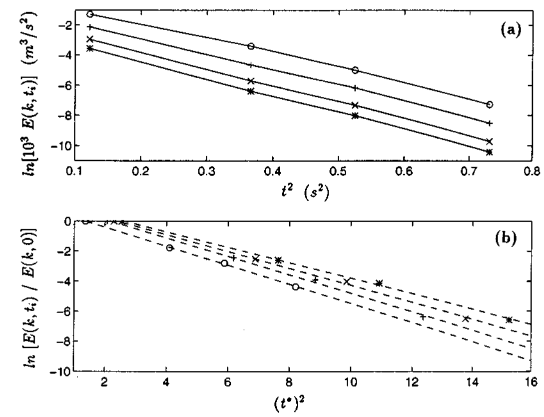

FIG. 3. Example of mode evolution in (a) dimensional and (b) nondimensional forms. The fits of (b) have slopes $\simeq -0.62$ [lower line and $k = 30$ $(2\pi/D)$ and $\simeq -0.5$ [upper line and $k = 130$ $(2\pi/D)$].

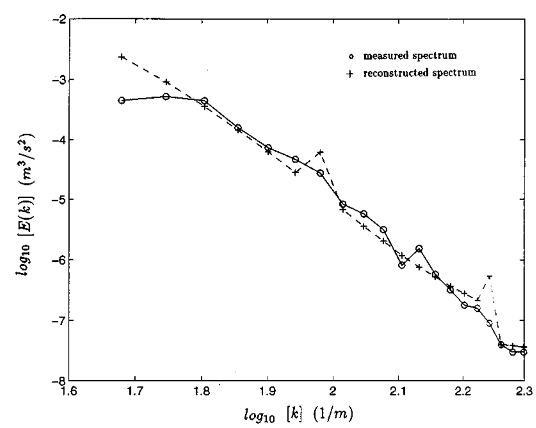

FIG. 4. Comparison between reconstructed and measured spectra for x_4 and $V_0 = 2.5$ m/s. The error committed in terms of distance from the jet orifice is about $4D$.

evolution law were obtained in another previous analysis [6], but considering scalar passive fields. In the present case our results appear to be more general since the velocity represents an active vector field. In any case, the results achieved so far indicate that the evolution law of the mode amplitude is the effect of the interaction between the background turbulence and the perturbation. Referring to Eq. (2), it is found that $G_1(t_i,k)\neq G_2(t_i,k)$, where $G_2(t_i,k)$ is correctly represented by Eq. (1) for $k>k_0$. As pointed out by Kraichnan [4], $G_2(t_i,k)$ is affected by the large-scale properties of the turbulent flow. The interactions between $u_{p2}(x_i,t)$ and $u_j(x_i,t)$ are in fact retained in the exponential law quadratic in t_i that reflects the Gaussian form of the probability distribution function of the large-scale fluctuations [4] of $u_j(x_i,t)$.

The question that arises is whether $G_2(t_i,k)$, approximated by Eq. (1), can be assumed to be the response function of the turbulent system. This point can be addressed by verifying its predictive properties. It is possible in fact to reconstruct the spectra at each position starting with a k^{-2} spectrum at $t=0$ and using Eq. (1). The predictive reliability can be validated by estimating the time delay needed to reconstruct the spectra measured at each point x_i and comparing it with the measured time delays t_i. On the other hand, one can calculate the time (or space) delay needed for the measured spectrum at x_i to reach, moving backward, the k^{-2} shape expected at $x=0$. Also in this case, if Eq. (1) is correct, the reconstructed time delay should be equal t_i. In Fig. 4 we present a result of the first approach. The reconstructed spectrum at x_4 and $V_0=8$ m/s is superposed with the measured one. The agreement between the measured and reconstructed time delay was good with an error that, in terms of space, was on the order of $4D$ downstream from the big jet. The same results are obtained also at the other x_i and with the other approach with an error of the same order. This systematic error of about $4D$ downstream is related to the fact that the background turbulence is not significant at small distances from the jet belonging to the laminar region of the jet flow, confirming that the perturbation is deformed only by diffusion in laminar regions.

In conclusion, results obtained from the experimental analysis of the interaction between a step-function perturbation and a turbulent velocity field have been reported. Measurements have been conducted in four positions downstream of the jet orifice by single probe hot wire anemometry. The analysis has been performed in the wavenumber domain, by the use of a local Taylor hypothesis, and in the time domain. Depending on the relative turbulence level of the background flow, the analyzed flow conditions were classified as laminar or turbulent. In the laminar case it was found that the dynamics is characterized by pure diffusive effects with diffusivity ν. In the turbulent experiments a proper synchronized averaging technique permits the extraction of the perturbation from the background turbulence. In this way the evolution of the perturbation interacting with the background turbulence has been analyzed and the response function of the turbulent system has been evaluated by the estimation of each mode evolution law. It is found that the time evolution predicted by the linear approximation of Kraichnan's DIA response function is in good agreement with present results. Physically this corresponds to a *ballistic* behavior of the velocity front that is related to the large-scale fluctuations of the background turbulence. The effect of such fluctuations is in fact retained by a response function that conserves a Gaussian functional form. These results have been checked by analyzing two different mean velocities and two different sizes of the small jet that generates the perturbation. The predictive properties of the linear response function are confirmed.

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[1] L.D. Landau and E.M. Lifshitz, *Fluid Mechanics*, 2nd ed. (Pergamon, Oxford, 1987).
[2] C.C. Lin, Proc. Symp. Appl. Math. (Am. Math. Soc.) **4** (1951).
[3] A.A. Townsend, *The Structure of Turbulent Shear Flows* (Cambridge University Press, Cambridge, 1956).
[4] R. Kraichnan, J. Fluid Mech. **5**, 497 (1959).
[5] M.M. Stanisic, *The Mathematical Theory of Turbulence*, 2nd ed. (Springer-Verlag, Berlin, 1988).
[6] S. Grossmann and I. Procaccia, Phys. Rev. A **29**, 1358 (1984).
[7] R.M. Kellog and S. Corrsin, J. Fluid Mech. **96**, 641 (1980).
[8] E.C. Itsweire and C.W. Van Atta, J. Fluid Mech. **145**, 423 (1984).
[9] M. Caponeri and S. Ciliberto, Physica D **58**, 365 (1992).
[10] A. Kolmogorov, C. R. Akad. Sci. SSSR **30**, 301 (1941).
[11] J.F. Pinton and R. Labbé, J. Phys. (France) II **4**, 1461 (1994).
[12] X. Waitz *et al.*, Phys. Rev. Lett. **63**, 1747 (1989).
[13] A. Papoulis, *Probability, Random Variables and Stochastic Process* (MIT Press, Cambridge, MA, 1984).